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Abstract

In the linear coinsurance problem, examined first by Mossin (1968), a higher risk aversion with respect to wealth in the sense of Arrow-Pratt implies a higher optimal coinsurance rate. We show that this property does not hold for health insurance under ex post moral hazard, i.e., when illness severity cannot be observed by insurers and policyholders decide on their health expenditures. The optimal coinsurance rate trades off a risk sharing effect and an incentive effect, both related to risk aversion. JEL Codes: D1, D8, I1. Keywords: Health insurance; ex post moral hazard; coinsurance.

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1 Introduction

The linear coinsurance problem, originally examined by Mossin (1968), plays an important role in the analysis of economic and financial decisions under risk, and this is for at least two reasons. Firstly, this model is suitable for tractable comparative statics analysis, in order to study wealth and income effects on insurance demand in various settings (e.g., with or without background risk, in a static or dynamic setting, etc...). Secondly, its conclusions can be straighforwardly adapted to the analysis of static portfolio choices when agents can invest in one risk-free asset and in one risky asset. An important property of this model states that the individual's degree of risk aversion with respect to wealth in the sense of Arrow-Pratt goes hand in hand with a higher optimal coinsurance rate: more risk averse individuals choose a higher coinsurance rate.

In this note, we will show that this property does not hold for health insurance under ex post moral hazard. There is ex post moral hazard in medical insurance when insurers do not observe the severity of illness and policyholders may exaggerate their health care expenses - Arrow (1963), Pauly (1968) and Zeckhauser (1970). Linear coinsurance under ex post moral hazard (i.e., when insurers pay the same fraction of the health care cost whatever the individuals' expenses) has been considered by many authors, including Zeckhauser (1970), Feldstein (1973), Arrow (1976), and Feldman and Dowd (1991) to analyze the trade-off between two conflicting objectives: providing risk coverage on one side, and incentivizing policyholders to moderate their health expenses on the other side.

In order to show that ex post moral hazard breaks the link between the degree of risk aversion and the optimal coinsurance rate, we will proceed through a simple example. We will consider a model where utility depends on wealth and health in an additive way, with constant absolute risk aversion with respect to wealth, and where the utility derived from health is linear. Furthermore, the only private information of individuals is about the severity of their illness. All other preference parameters, including health risk exposure and risk aversion are either observed by insurers, or rather recovered from observable variables such as age, education, occupation, marital status or from past loss experience. These very crude assumptions are obviously not chosen for the sake of realism, but because they allow us to focus on the ex post moral hazard problem in a fully computable model, without interfering with adverse or advantageous selection issues. It will turn out that, in this model, the optimal coinsurance rate does not depend on the index of absolute risk aversion.

The intuition for this result goes through two effects of an increase in the coinsurance rate. On one hand, for a given pattern of health care expenses, a larger coinsurance rate offers a better risk protection to risk averse individuals: thus, the larger the degree of risk aversion, the larger the benefit drawn for this more complete risk coverage. This is the standard channel that links together the intensity of risk aversion and the optimal insurance coverage. On the other hand, an increase in coverage exacerbates health care overexpenses, and it turns out that, in an expected utility setting such as ours, this (dis)incentive effect is the larger as the absolute risk aversion is small. Thus, more risk aversion entails less financial risk, because it corresponds to less health care overexpenses, and thus it reduces the need for insurance coverage. When the index of absolute risk aversion increases, the risk protection effect and the incentive effect push the optimal coinsurance rate upwards and downwards, respectively. In the model that we will consider, these two effects exactly balance each other out, so that the optimal coinsurance rate remains unchanged.

2 A computable example

Let us consider an individual whose welfare depends both on monetary wealth R and health level H, with a separable bi-variate von Neumann-Morgenstern utility function $U(R, H) = u(R) + v(H)$. The individual displays CARA preferences with respect to wealth, i.e., $u(R) = -\exp\{-\alpha R\}$, where α is the index of absolute risk aversion, and $v(H) = \beta H, \beta > 0$. We thus have

$$U(R, H) = -\exp\{-\alpha R\} + \beta H.$$

Health may be negatively affected by illness, but it increases with the health care expenses. This is written as

$$H = h_0 - \gamma x(1 - m), \gamma > 0,$$

where h_0 is the initial health endowment, x is the severity of illness and m is the health care expense level. Illness severity is distributed as a random variable X over an intervall $[a, b]$,with $a > 0$, and the parameters of the problem are such that $m \in [0,1]$. Thus, the health level H increases linearly from $h_0 - \gamma x$ to h_0 when m increases from 0 to 1.

The individual's insurance contract specifies that a fraction θ of the monetary expenses are reimbursed and that the insurance premium P is actuarial. It is assumed that insurers observe all the characteristics of insurance seekers, including their risk exposure and degree of risk aversion (i.e., the probability distribution of X and parameter α). In more concrete terms, insurers are supposed to be able to recover these information through observable characteristics, such as age, gender or level of education.[1]

In state x, the individual's wealth is

$$R(x) = w - (1-\theta)m(x) - P,$$

where $m(x)$ denotes the health care expenses in state x. The individual chooses $m(x)$ such that

$$m(x) \in \arg\max_{\widehat{m}\in[0,1]} [-\exp\{-\alpha(w - (1-\theta)\widehat{m} - P)\} + \beta\gamma x\widehat{m}].$$

Let us assume that $m(x) \in (0,1)$ for all x. Later, we will find conditions under which this is actually the case. Then, the first-order condition for $m(x)$ to be an optimal choice of the individual is written as

$$-\alpha(1-\theta)\exp\{-\alpha R(x)\} + \beta\gamma x = 0,$$

which implies

$$R(x) = \frac{1}{\alpha}\ln\left[\frac{\alpha(1-\theta)}{\beta\gamma x}\right], \tag{1}$$

and

$$m(x) = \frac{\alpha(w-P) + \ln\left[\frac{\beta\gamma x}{\alpha(1-\theta)}\right]}{\alpha(1-\theta)}.$$

Using $P = \theta E[m(X)]$ yields

$$\begin{aligned} E[R(X)] &= w - (1-\theta)E[m(X)] - P = w - \frac{P}{\theta} \\ &= \frac{1}{\alpha}E\left[\ln\left(\frac{\alpha(1-\theta)}{\beta\gamma X}\right)\right]. \end{aligned}$$

[1] Outreville (2014) surveys the empirical analysis of socio-demographic variables associated with risk aversion.

which gives

$$P = \theta w + \frac{\theta}{\alpha} E\left[\ln\left(\frac{\beta\gamma X}{\alpha(1-\theta)}\right)\right], \tag{2}$$

and using (1) and (2) yields

$$\begin{aligned} m(x) &= \frac{w - P - R(x)}{1-\theta} \\ &= w - \frac{1}{\alpha}\ln[\alpha(1-\theta)] + \frac{\ln(\beta\gamma x) - \theta E[\ln(\beta\gamma X)]}{\alpha(1-\theta)} \end{aligned} \tag{3}$$

By disregarding the constant term $h_0 - \beta\gamma E[X]$, (1) and (3) allow us to write the individual's expected utility as

$$\begin{aligned} & -E\left[\exp\{-\alpha R(X)\}\right] + \beta\gamma E[Xm(X)] \\ = & -\frac{\beta\gamma E[X]}{\alpha(1-\theta)} \\ & +\beta\gamma\left[E[X]w - \frac{E[X]}{\alpha}\ln[\alpha(1-\theta)] + \frac{E[X\ln(\beta\gamma X)] - \theta E[X]E[\ln(\beta\gamma X)]}{\alpha(1-\theta)}\right], \end{aligned}$$

which is maximized with respect to $\theta \in [0,1]$. Let $z = 1/(1-\theta)$. Equivalently, z maximizes

$$V(z) \equiv E[X]\ln(z) + z[\Delta - E[X]],$$

in $[1, +\infty)$, where

$$\begin{aligned} \Delta &= E[X\ln(X)] - E[X]E[\ln(X)] \\ &= cov[X, \ln(X)] > 0. \end{aligned}$$

We have

$$\begin{aligned} V'(z) &= \Delta - E[X] + \frac{E[X]}{z}, \\ V''(z) &= -\frac{E[X]}{z^2} < 0. \end{aligned}$$

and

$$V'(1) = \Delta > 0$$

If

$$\Delta < E[X], \tag{4}$$

then $V(z)$ is maximized over $[1, +\infty)$ when

$$z = \frac{E[X]}{E[X] - \Delta} > 1,$$

that is

$$\theta = \frac{\Delta}{E[X]} = \frac{cov[X, \ln(X)}{E[X]} \in (0, 1). \tag{5}$$

If $\Delta \geq E[X]$, then $\theta = 1$ would be an optimal corner solution, with $m(x) = 1$ for all x. Thus (4) is a necessary condition for an optimal interior solution to exist. (3) shows that $m(x)$ is increasing for such an interior solution. Thus we have $m(x) \in (0, 1)$ for all $x \in [a, b]$ if

$$w \in (\underline{w}, \overline{w}), \tag{6}$$

where $\underline{w}$ and $\overline{w}$ are given by (3), $m(a) = 0, m(b) = 1$ and $\theta = \Delta/E[X]$, with $\overline{w} > \underline{w}$ if

$$\ln(b/a) < \alpha(1 - \theta). \tag{7}$$

In short, under (4),(6) and (7), we have an interior optimal solution $\theta = \Delta/E[X] \in (0, 1)$ with $m(x) \in (0, 1)$ for all x. At this optimal solution, the coinsurance rate θ is given by (5) and it is independent from the index of absolute risk aversion α. It only depends on the probability distribution of the illness severity X.[2]

3 Comments

Let $R^*(x)$ be the individual's wealth that would be chosen in state x in the absence of insurance, that is

$$R^*(x) = \frac{1}{\alpha} \ln \left[\frac{\alpha}{\beta \gamma x} \right],$$

[2]Everything else given, (7) does not hold when α is small enough. In that case, $m(x)$ is equal to 0 or 1 in a sub-interval of $[a, b]$. Thus, strictly speaking, the independence of θ from α has been established among values of α that are large enough for such corner solutions no to be optimal.

with $R(x) < R^*(x)$ when $\theta > 0$. $R^*(x)$ is ex post efficient, since the marginal utility of wealth is the same, be it used for health care or for other uses. A *decrease* in $R(x)/R^*(x)$ corresponds to an *increase* in health care overexpenses (i.e., an increase above the ex post efficient level).[3] Assume $R^*(x) > 0$, and thus $\ln(\alpha/\beta\gamma x) > 0$. We have

$$\begin{aligned} \frac{\partial[R(x)/R^*(x)]}{\partial\theta} &= -\frac{1}{(1-\theta)\ln(\alpha/\beta\gamma x)} < 0, \\ \frac{\partial^2[R(x)/R^*(x)]}{\partial\theta\partial\alpha} &= \frac{1}{\alpha(1-\theta)\ln(\alpha/\beta\gamma x)^2} > 0. \end{aligned}$$

Thus, as expected, an increase in θ increases the health care overexpense (i.e., $R(x)/R^*(x)$ decreases), but the larger the index of risk aversion, the smaller this induced expense effect. In other words, more risk aversion corresponds to less financial risk, because it corresponds to less health care overexpenses. Coming back to our initial notation, the index of risk aversion is equal to $-d[\ln(u'(R)]/dR$, and thus it measures the rate of increase in the marginal utility of wealth when wealth decreases, and thus it is not astonishing that the larger the absolute risk aversion, the smaller the insurance-induced increase in health care spending. This induced risk exposure is anticipated by insurance seekers, and thus it is a reason why more risk averse individuals may purchase less insurance. Conversely, for a given risk exposure, more risk averse individuals tend to purchase more insurance. In the absence of loading, they would purchase full coverage if there were no ex post moral hazard. In the present model, the two mechanisms compensate exactly, and ultimately risk aversion does not affect the optimal coinsurance rate.

4 Conclusion

Risk aversion may depend on several parameters, including wealth, age, marital status and occupation, among others. Consider the case of a background risk, such as business interruption, assumed to be uninsurable and in force for self-employed people, but not for employees. Under risk vulnerability, such a background risk makes the individual more averse to other independent risks, including health care expenditures. If insurance expenses were

[3] Note that $R(x)/R^*(x)$ is decreasing with respect to x. Thus, the larger the severity of illness, the larger the distortion in health expenses due to ex post moral hazard.

perfectly monitored by the insurer, then this background risk would increase the coinsurance rate for health care. In other words, everything else given, self-employed people should choose a more complete health insurance than employees. The previous example shows that this is no more the case under ex post moral hazard.

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